Chapter 2

Physics motivation

2.1 Current knowledge of neutrinos

The existence of the neutrino was postulated in 1930 by W. Pauli[1] to explain the apparent energy nonconservation in nuclear weak decays. It was another 23 years before this bold theoretical proposal was verified experimentally in a reactor experiment performed by C. Cowan and F. Reines[2]. The most fundamental properties of the neutrino were verified during the subsequent decade. The neutrino was shown to be left handed in an ingenious experiment by Goldhaber, Grodzins and Sunyar[3] in 1957. The distinct nature of ν_e and ν_μ was demonstrated in 1962 in a pioneering accelerator neutrino experiment at Brookhaven by Danby *et al.*[4].

The following years saw remarkable progress in neutrino experiments, especially those utilizing accelerators as their sources. Increases in available accelerator energies and intensities, advances in neutrino beam technology, and more sophisticated and more massive neutrino detectors were all instrumental in our ability to perform ever more precise neutrino experiments. The focus of those experiments, however, was until very recently mainly on using neutrinos as a probe in only two areas. First, together with experiments utilizing electrons and muons, the worldwide neutrino program played a key role in measuring the nucleon structure functions. Second, along with a variety of other efforts (especially e^+e^- annihilations, inelastic electron scattering), neutrino experiments played a key role in establishing the validity of the Standard Model, through the discovery of neutral currents[5], measurements of the neutral-current to charged-current ratio[6], and measurements of the neutrino lepton scattering cross sections[7].

We are entering now a new era in experimental neutrino physics whose main thrust will likely be twofold: better understanding of the nature of the neutrino, i.e., a study of the neutrino properties, and use of the neutrino in astrophysics and cosmology as an alternative window on the universe, to supplement investigations with electromagnetic radiation. The MINOS experiment, which addresses the subject of neutrino oscillations, will make important contributions to the first part of this program.

Neutrinos are among the fundamental constituents in nature. The space around us is permeated with neutrinos which are relics of the Big Bang, with about 110 ν's/cm^3 for every neutrino flavor. But our knowledge of the neutrino's properties lags far behind our knowledge of other elementary constituents, for example, the charged leptons. A few examples will illustrate this point, where we quote the lepton values from the latest compendium by the Particle Data Group[8]:

- We do not know whether neutrinos have mass; our current information gives us only upper limits ranging from a few eV for ν_e to some 20 MeV for ν_τ. We can contrast that with a fractional mass error of about 3×10^{-7} for the electron and muon and about 2×10^{-4} for the tau.

- We do not know if neutrinos are stable or decay, either into neutrinos of other flavors or into some new, as yet undiscovered, particles. In contrast, we know that the electron is stable, and we know the muon lifetime with a fractional error of 2×10^{-5} and the tau lifetime at the level of 0.5%.

- Finally, we do not know if the neutrinos have electromagnetic structure, for example a magnetic moment. The electron magnetic moment is known with a precision of about one part in 10^{11}, and the magnetic moment of the muon to one part in 10^8.

These are only a few examples of our ignorance of the basic nature of neutrinos, but they are sufficient to demonstrate that almost half a century after their discovery, neutrinos are still poorly understood. Because of their fundamental nature, we cannot profess to understand our universe without understanding neutrinos.

2.2 Neutrino masses and oscillations

The study of neutrino oscillations offers us potentially the most sensitive means to search for and to measure neutrino masses (or, to be precise, neutrino mass-squared differences). Observation of a nonzero neutrino mass, which would follow directly from the observation of neutrino oscillations, would be a clear example of a breakdown of the Standard Model and thus an indication of physics beyond it. Many of the popular extensions of the Standard Model do indeed predict nonzero neutrino masses and the existence of neutrino oscillations[9]. Furthermore, neutrino oscillations are more than just an attractive theoretical concept: the existence of the phenomenon is strongly suggested by several experimental observations:

a) The need for dark (i.e., non-shining) matter[10], is based mainly on three phenomena: the motion of galaxies within clusters of galaxies, the flat rotational curves for stars in spiral galaxies, and the successes of inflationary Big Bang cosmology which predicts that the density of the universe equals the so-called critical density. Neutrinos, since they are present in abundance everywhere, could account for at least a part of the dark matter if they have finite mass.

b) The solar neutrino deficit, i.e., the observation of fewer sun-originated neutrinos on earth than is expected from the known solar luminosity[11].

c) The atmospheric neutrino anomaly[12], i.e., a measured ν_μ/ν_e ratio for neutrinos from cosmic ray interactions in our atmosphere which is significantly smaller than predicted.

The hypothesis that this anomaly is caused by neutrino oscillations is strongly supported by the recent observation of an up-down asymmetry in the atmospheric ν_μ flux by the Super-Kamiokande Collaboration[13], as well as by their studies of upward going muons.

d) The apparent observation of $\overline{\nu}_e$ in an almost pure $\overline{\nu}_\mu$ beam in the Los Alamos LSND experiment[14].

The MINOS experiment can explore a large region in oscillation parameter space. Furthermore, it can confront directly and conclusively the atmospheric neutrino anomaly and should be able to check the validity of the oscillation interpretation for the LSND effect. In the discussion in Section 2.3, which describes these hints in more detail, we shall emphasize the current status of the atmospheric neutrino anomaly. But first we shall describe briefly the standard neutrino oscillation formalism.

The underlying principle behind neutrino oscillations[15] is the fact that, if neutrinos have mass, then a generalized neutrino state can be expressed either as a superposition of different mass eigenstates or of different flavor eigenstates. This is mainly a restatement of a well known quantum mechanics theorem that, in general, several different basis vector representations are possible, with the different representations being connected by a unitary transformation. Other well known examples of this principle in particle physics are the $K^o/\overline{K}^o$ system (strong interaction and weak interaction eigenstates) and the quark system (weak interaction and flavor eigenstates connected by the CKM matrix).

From the study of e^+e^- annihilations at the Z^o peak[16], we know that there are only three light neutrino flavor eigenstates. Accordingly, the most likely situation is that we have three mass eigenstates and that the connecting unitary matrix is a 3×3 matrix. This is not rigorously required since we could have states with $m_\nu > m_Z/2$ or flavor states that do not couple[17] to the Z^o. Even though such possibilities appear *a priori* unaesthetic, there has recently been significant theoretical effort to see whether such mechanisms could explain some of the anomalous effects seen in neutrino experiments.

Thus, for the 3-flavor case, the weak eigenstates $|\nu_a\rangle = \nu_e, \nu_\mu, \nu_\tau$ and the mass eigenstates $|\nu_i\rangle = \nu_1, \nu_2, \nu_3$ are related by

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = [U] \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} \tag{2.1}$$

i.e., $\nu_\alpha = U\nu_i$, where U is the unitary matrix that can be parametrized as (in analogy with the CKM matrix):

$$U = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13} & C_{12}C_{23} - S_{12}S_{23}S_{13} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13} & -C_{12}S_{23} - S_{12}C_{23}S_{13} & C_{23}C_{13} \end{bmatrix} \tag{2.2}$$

where $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$ and for simplicity we have taken the phase $\delta = 0$, i.e., assumed CP conservation.

The probability, then, that a state, which is pure ν_α at $t = 0$, is transformed into another flavor β at a time t later (or distance L further), is

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}\sin^2\left(\frac{\Delta m^2_{ij}L}{2E}\right), \tag{2.3}$$

with E being the energy of the neutrino and

$$\Delta m^2_{ij} = m^2(\nu_i) - m^2(\nu_j). \tag{2.4}$$

Thus (assuming CP invariance) we have five independent parameters: three angles, θ_{12}, θ_{23}, and θ_{13} and two Δm^2_{ij} (the third Δm^2_{ij} must be linearly related to the first two). All of the neutrino oscillation data must then be capable of being described in terms of these five parameters. Furthermore, if at least two neutrinos have nondegenerate, nonzero masses and if the mixing angles are nonzero, neutrino oscillations must exist.

Clearly, the above expression is complicated and the relationship of experimental results to the five basic parameters somewhat obscure. It also could be that Nature has arranged itself in such a way that this full 3×3 formalism is not required, at least to explain the currently available data, and that a two flavor approximation is adequate.

As a minimum such a two-flavor representation provides a much easier way to parametrize the existing and expected future data. In addition, it would be a good approximation if the matrix U has similar structure to the CKM matrix (i.e., is almost diagonal). In this formalism it is customary to represent the results of a single experiment in terms of oscillation between two flavors and involving only two mass eigenstates, hence only one Δm^2_{ij}. The two possible representations of a given neutrino state are then related by

$$\begin{bmatrix} \nu_\alpha \\ \nu_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}. \tag{2.5}$$

This approximation yields the well known transition probability equation

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \;\sin^2\left(1.27\Delta m^2 \frac{L}{E}\right), \tag{2.6}$$

giving the probability of conversion of a neutrino of energy E and flavor α into a neutrino of flavor β after traversing a distance L, where L is in km (m), E in GeV (MeV), and $\Delta m^2 = m_1^2 - m_2^2$ in eV^2. This expression is obviously much simpler than the one for the three flavor case and the results of any experiment, within the framework of this formalism, can be easily displayed on a two-dimensional plot since only two physics parameters, θ and Δm^2, are involved.

2.3 Hints for neutrino oscillations

In Section 2.2 we enumerated briefly the current hints for neutrino oscillations. In the present Section we shall elaborate on this topic in more detail, emphasizing especially the results of atmospheric neutrino measurements, since it is these results that are most germane to MINOS. Figure 2.1 summarizes the current picture of positive evidence for neutrino oscillations. In this plot, we take at face value the exclusive limits presented by the relevant experiments with negative results.

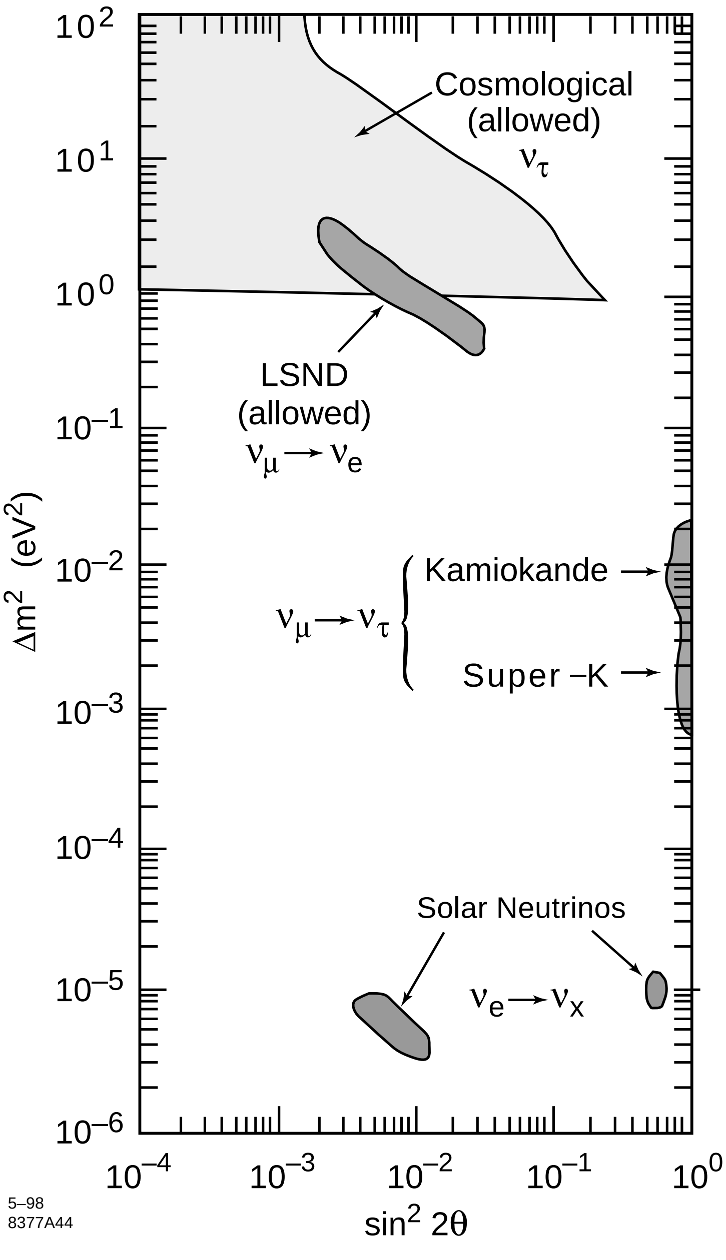

Figure 2.1: Regions of neutrino oscillation parameter space in which positive experimental evidence exists for neutrino oscillations.

We can make several observations about Figure 2.1. First of all, the three sets of experimental results (solar neutrino deficit, atmospheric neutrino anomaly, LSND effect) indicate three different mass-squared scales: approximately 10^{-5}, 10^{-3} to 10^{-2} and 1 eV^2, respectively. As mentioned in Section 2.2, this is incompatible with the conventional picture of three species of neutrinos. One can resolve this difficulty by invoking a fourth, sterile neutrino. Alternatively, at least one of the three sets of data would have to be wrong or require an explanation outside of the area of neutrino oscillations.

The next observation concerns the atmospheric neutrino anomaly. The contour plots shown indicate the best currently available analyses of all the data on zenith angle distributions and the ν_μ/ν_e ratio, from both Kamiokande and Super-Kamiokande. The results of the two experiments give only a small region of overlap in parameter space.

The third point has to do with the LSND effect. The Figure shows only that small part of the LSND-suggested parameter space which is not incompatible with the results of other, negative result, experiments.

Finally, the region in Figure 2.1 indicated by dark matter evidence is somewhat arbitrary and not directly comparable to the three sets of experimental data. The missing dark matter arguments give indications of possible masses of neutrinos, not of their mass squared differences. But if the idea of neutrino mass hierarchy is valid, then this comparison is justified. In general, cosmological mass arguments suggest neutrino masses in the range of a few to a few tens of eV, and do not say anything about mixing angle. Thus, very conservatively, we indicate the suggested region as above 1 eV^2 and cut off the large $\sin^2(2\theta)$ region on the right using the limits from the most recent $\nu_\mu \to \nu_\tau$ oscillation laboratory experiments.

We proceed now to discuss these four pieces of evidence in more detail.

2.3.1 The dark matter issue

The dark matter topic is complex, from both theoretical and observational points of view, and we can give only a very brief discussion of the subject in this document. In addition, it is unlikely that the MINOS experiment will confront this particular area of the physics of neutrino masses. That possibility is not completely excluded, however: if two mass states are relatively heavy (in the few eV range) but almost degenerate, i.e., Δm^2 in the 10^{-3} to 10^{-1} eV^2 range, then MINOS would be sensitive to oscillations between these two massive states. In this Section we limit ourselves to just a brief summary of the most pertinent facts and ideas.

Probably the most significant piece of relevant evidence in this area comes from the measurements of rotation velocities of stars in spiral galaxies, which indicate that these velocities stay constant out to very large distances. From simple mechanics, this implies the existence of mass at large radii in amounts significantly larger than accountable by the observed "shining" matter. There are many candidates for this dark matter: their mass spectrum extends from some 10^{-5} eV for the axion hypothesis to about 10^4 solar masses for the black hole hypothesis – a range of masses of some 75 orders of magnitude. Clearly, this topic is still quite speculative[18].

Cosmological models in which neutrinos provide all the missing mass needed to close the universe call for a neutrino mass of about 30 eV. Aside from other problems with these models, such neutrinos could not account for all the dark matter in spiral galaxies because the Pauli exclusion principle limits their number and thus requires a mass exceeding 80 eV[18, 19]. The currently favored view is that dark matter is composed of a number of different components, massive neutrinos possibly being one of them.

This cosmological dark-matter motivation for nonzero mass neutrinos has led to the initiation of a short baseline neutrino oscillation search program at CERN, aimed at detection of ν_τ, with two experiments, CHORUS and NOMAD. The data taking phase has been completed (possibly NOMAD may run one more year) and initial results from the analyses have already been reported[20, 21]. The best limits on $\sin^2(2\theta)$ for massive ν_τ (responsible for the cutoff of the cosmologically interesting region on the right in Figure 2.1) come from these experiments.

2.3.2 LSND effect

The LSND Collaboration has published evidence for a $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ transition from an experiment at LAMPF[22]. The data were taken in an experiment where π^+'s produced in a water target by the 800 MeV primary protons were stopped in a downstream copper beam stop. The resulting neutrinos, both from π^+ and μ^+ decays, were then detected in a large liquid scintillator tank. Experimental conditions were such that neutrinos from π^- and μ^- (and hence any primary $\overline{\nu}_e$ component) were suppressed by more than a factor of 10^3.

The experimental signature of a $\overline{\nu}_e$ reaction:

$$\overline{\nu}_e + p \rightarrow e^+ + n$$

was correlated signals, in space and time, from the primary positron and the delayed gamma ray from subsequent neutron capture. Cosmic ray background was measured with data taken during the beam off part of the machine cycle. The published analysis yielded 22 candidate events with the expected background of 4.6 ± 0.6. The measured oscillation probability for $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ was $\mathrm{P} = (0.31 \pm 0.12 \pm 0.05)\%$.

The resulting contours in oscillation parameter space, at both 90% and 99% confidence levels, are shown in Figure 2.2. Also shown are the excluded regions from the negative results of several other experiments, which apparently rule out a large fraction of the LSND-suggested region. The CCFR[23] and, more recently, NOMAD[21] experiments exclude most of the high mass region, KARMEN[24] and BNL E-776[25] the intermediate region, and the Bugey reactor[26] experiment the lowest Δm^2 region. The LSND "sliver" shown in Figure 2.1 represents that part of the LSND region which is compatible with all of those experiments. The LSND data cannot be used to determine a unique set of oscillation parameters for this allowed region because the L/E range is not very large and its value is not determined very precisely on an event by event basis.

The LSND collaboration is continuing to take data, and with the new data they should be able to improve their statistics with somewhat different systematics. In addition, the Collaboration has analyzed the decay-in-flight data, which are sensitive to $\nu_\mu \rightarrow \nu_e$ oscillations with the ν_μ from π decay in flight[27]. Their analysis of these data is consistent with the published results but the significance is weaker and one is not able to determine the oscillation parameters any better.

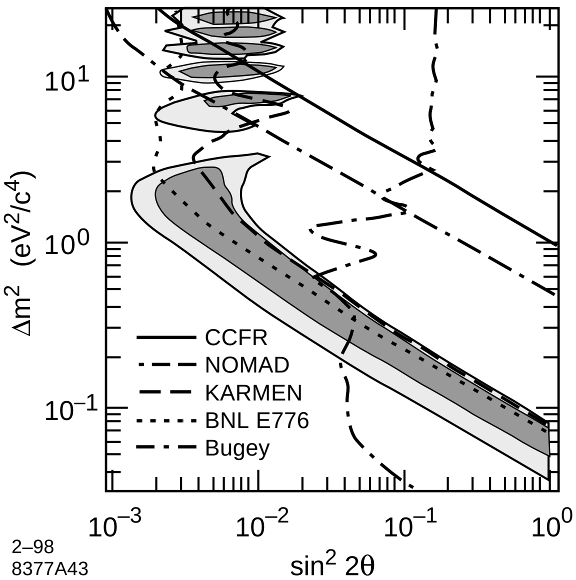

Figure 2.2: Regions of neutrino oscillation parameter space in which $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ oscillations are suggested by the LSND experiment.

The experiment which is able to confront the LSND results most directly is KARMEN[24], which uses neutrinos from the spallation source ISIS at the Rutherford Appleton Laboratory. This experiment is very similar to LSND in the reaction studied, the source of neutrinos, and the general method of detection. Its sensitivity is lower by a factor of 2 to 3 because of its higher backgrounds, smaller detector mass and the shorter source to detector distance. However the fine grained segmentation of their detector and the excellent L/E determination (2 to 3%) for each event partially offsets the more negative features. KARMEN is just commencing a new run, with a much better shielding arrangement which has significantly reduced the background from cosmic-muon produced neutrons. It was those neutrons which limited the sensitivity of the original run. Indications from the initial data taken under the changed conditions are that the improved shielding works as well as expected and that the experiment should be able to cover the LSND region completely[28].

Recently a new proposal, MiniBooNE[29], has been submitted to Fermilab with a goal of investigating the LSND effect with a new detector, similar to LSND, exposed to neutrinos from the Fermilab 8-GeV Booster. Their sensitivity, based on Monte Carlo calculations, is claimed to be roughly a factor of 3 to 5 better than LSND. The MiniBooNE proposal has been given Stage I approval by the Fermilab Director following the May 1998 Fermilab PAC meeting.

2.3.3 The atmospheric neutrino anomaly

It is this specific neutrino puzzle that MINOS can confront most directly and thus the situation here is most relevant to the potential physics of the MINOS experiment. Accordingly we shall give a more detailed description of the current status of this anomaly.

The cosmic rays (protons or heavier nuclei), which impinge on our atmosphere from above, will generally interact in its first 100 g or so, i.e., in the top 10% of the atmosphere. These interactions will ordinarily produce a number of π's and K's, which subsequently will either interact themselves, thus continuing the hadronic cascade, or decay. These first few interactions, which produce most of the secondary hadrons, generally occur far enough from the earth's surface that most of the muons resulting from π or K decay will themselves decay before hitting the earth. The net effect is that the ratio of ν_μ's to ν_e's arriving at the earth, in the 1 GeV range and below, will be close to a factor of 2 (one of each flavor from muon decay, and one ν_μ from hadron decay).

There are now a number of theoretical calculations which attempt to perform rather detailed and realistic calculations of this ratio as a function of neutrino energy, putting in all that is known about the relevant physical phenomena: cosmic ray composition and spectra, the evolution of hadronic cascades, geomagnetic field effects, and the exact nature of pion, kaon, and muon decays[30]. These calculations predict both the absolute values of the fluxes and the ratio of neutrino flavors. They tend to reproduce very closely the result of the above simple argument and find that, even though there is an uncertainty of about $\pm 20\%$ in the absolute normalization of the neutrino flux, the flavor ratio calculation is good to better than $\pm 5\%$.

Several experiments have now studied this ratio and generally find a deficiency of muon neutrinos, the so called "atmospheric neutrino anomaly". The experiments can be conveniently classified into two categories: those that use large water Cerenkov counters and those that use solid media instrumented with gas chambers. The purely experimental systematics in these two sets of detectors should be quite different. We discuss the results from these two sets of experiments in the next two Sections.

2.3.3.1 The results from water Cerenkov counters

The initial studies of atmospheric neutrinos were performed by the IMB (Irvine-Michigan-Brookhaven) Collaboration[31] and the Kamiokande Collaboration[32]. Both detectors were located deep underground; both were initially motivated by the search for proton decay; both used ultra pure water as a neutrino target material and as the Cerenkov radiator. The photomultipliers viewed the volume from the inner surface of the detector. The Kamiokande experiment had the capability to veto cosmic ray muons by means of an optically isolated outer layer of water viewed by an independent set of photomultipliers. The IMB analysis relied on vetoing the throughgoing muons in software. It was the pattern of hits in the photomultipliers which allowed these detectors to distinguish between charged current ν_μ and ν_e interactions. The fact that this method works in the GeV range and below was verified by exposure of similar detector configurations at KEK to muon and electron beams from the KEK accelerator. The identification was shown to be good at the level of 99%[33].

Water Cerenkov experiments can make reasonably good estimates of electron neutrino energies from the total numbers of hits observed. On the other hand, for ν_μ's the events have to be classified into "fully contained events," where the muon stops in the detector, and the "partially contained, multi-GeV events," where the muon exits the detector. The total energy can be determined only for the fully contained events. The data are generally analyzed separately for the sub-GeV sample (low energy ν_e's and ν_μ's, namely those with $E_{vis} < 1.33$ GeV) and the multi-GeV sample (high energy ν_e's and ν_μ's, and exiting muon events). To eliminate the uncertainty in the absolute flux normalization, it is conventional to evaluate and present a ratio of ratios, R, defined by:

$$R = \frac{(\nu_\mu/\nu_e)_{data}}{(\nu_\mu/\nu_e)_{MC}}.$$

In addition, one can try to obtain some information about the L/E value of the observed events. To a good approximation one can deduce the value of L (flight path of the neutrino) from the measured zenith angle. However the correlation is such that for zenith angles of the order of 90°, a small error in the angle measurement gives a large error in the value of L. Furthermore, as mentioned above, one does not measure energy for all the events. Thus it is conventional to look at the flavor ratio and at the individual ν_μ and ν_e rates as functions of zenith angle for the two sets of events. Because of measurement errors, Fermi motion, and the finite momentum carried off by the unseen low energy particles, the zenith angle measurement improves at higher energies.

Both IMB and Kamiokande reported deficits of muon neutrinos, i.e., values of R below unity. In addition, the Kamiokande data showed a zenith angle dependence of the R value for the multi-GeV data set[34]. This effect, even though not statistically compelling, was in the direction which would be expected from the neutrino oscillation hypothesis, i.e., larger depletion of ν_μ's for the upward going direction. This angular dependence allowed one to set an upper limit of about 0.1 eV^2 on Δm^2. The sub-GeV data sample was consistent with no angular dependence[32, 34]. Neither of these experiments could provide any significant information on the neutrino oscillation mode, i.e., $\nu_\mu \leftrightarrow \nu_e$ *vs* $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_\mu \leftrightarrow \nu_{sterile}$.

Recently[13], the Super-Kamiokande Collaboration reported their analysis of the first 535 days of data taking from their new detector. Like the original Kamiokande detector, the new detector is also located in the Mozumi mine in Japan and is also a cylindrical water Cerenkov detector with an optically separated outer region used for anti-coincidence to eliminate cosmic ray muon background. The primary difference is the much larger fiducial mass of the Super-Kamiokande detector: 22.5 kt, about 20 times larger than Kamiokande or IMB. In addition to increasing the rate of atmospheric neutrino interactions in proportion to the fiducial volume increase, this larger size increases the fraction of contained ν_μ events.

At the present time the Super-Kamiokande detector has collected 33.0 kt-years of analyzed atmospheric neutrino data[13]. The global R values for both the sub-GeV and multi-GeV data sets are consistent with the Kamiokande values, as can be seen from Table 2.1.

	Sub-GeV		Multi-GeV	
Detector	Observed	Expected	Observed	Expected
Kamiokande	0.60±0.06±0.05	1.00	0.57±0.08±0.07	1.00
Super-Kamiokande	0.63±0.03±0.05	1.00	0.65±0.05±0.08	1.00

Table 2.1: Comparison of the Kamiokande[34] and Super-Kamiokande[13] R-value results. The ratios are calculated based on the Honda *et al.*[30] flux model. The first error shown for each ratio is from statistics and the second is from systematics.

In addition, there is a pronounced and statistically significant variation of the ν_μ rate as a function of the zenith angle, as shown in Figure 2.3(a), with the upward going ν_μ events being significantly depleted. This effect increases as the neutrino energy increases. In contrast, the ν_e distribution appears to be consistent with the Monte Carlo prediction if one allows for a 20% flux normalization uncertainty.

All of the available Super-Kamiokande data have been analyzed with programs similar to those used for the Kamiokande analysis[35]. The results of these analyses, for the $\nu_\mu \rightarrow \nu_\tau$ hypothesis, are shown in Figure 2.3(b). Based on these data, the Super-Kamiokande Collaboration has concluded that the data give evidence for neutrino oscillations in the modes $\nu_\mu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_{sterile}$, with $10^{-3} < \Delta m^2 < 10^{-2}$ eV^2 and $\sin^2(2\theta) > 0.8$.

2.3.3.2 Results from the solid gas-chamber detectors

The fine grained gas calorimeters which have contributed data relevant to the question of the atmospheric neutrino anomaly were also originally constructed to search for proton decay. Hence they were also located underground so as to provide adequate shielding against cosmic rays. The results from the first two of these detectors, Frejus[36] and NUSEX[37] indicated that the value of R is consistent with unity, i.e., no anomaly, even though the errors on these measurements were quite large. Accordingly, there was a question for some time whether the Kamiokande-IMB result could be caused by some instrumental effect. More specifically, some of the possible differences in the experimental setups that could be responsible for the difference in the results between water Cerenkov and iron calorimeter detectors are: difference in neutrino interactions in water and iron, different detection technique, different energy and spatial resolution, different methods of neutrino flavor determination, and different depths and hence different cosmic ray muon rates.

The importance of the Soudan 2 experiment is that it is able to confront these specific questions. Soudan 2 is a 1 kt iron TPC which produces event pictures of close to heavy-liquid bubble-chamber quality. Thus it is easy to distinguish track-like and shower-like events. At atmospheric neutrino energies, about two thirds of the events are quasi-elastic, containing only a single lepton plus a recoil proton. In Soudan 2 almost half of the recoil protons are visible as short low energy tracks whereas in water Cerenkov detectors most of the recoil protons are below Cerenkov threshold and hence invisible. The tracks and showers can be reconstructed in three dimensions. An ionization measurement is also obtained and, together with a Coulomb scattering measurement, allows one to to separate protons from muons.

Based on analysis of the current data, which corresponds to an exposure of 3.9 kt years[38, 39], the Soudan 2 Collaboration has reported a value of R of $0.64 \pm 0.11 \pm 0.06$. This result is completely compatible with, and hence confirms the existence of, the atmospheric neutrino anomaly first seen by the water Cerenkov experiments. The Soudan 2 data can provide a better determination of the L/E values for individual events because the detector measures recoil protons in some quasi-elastic interactions. The preliminary results of this analysis[39] favor values of Δm^2 somewhat higher than those suggested by Super-Kamiokande.

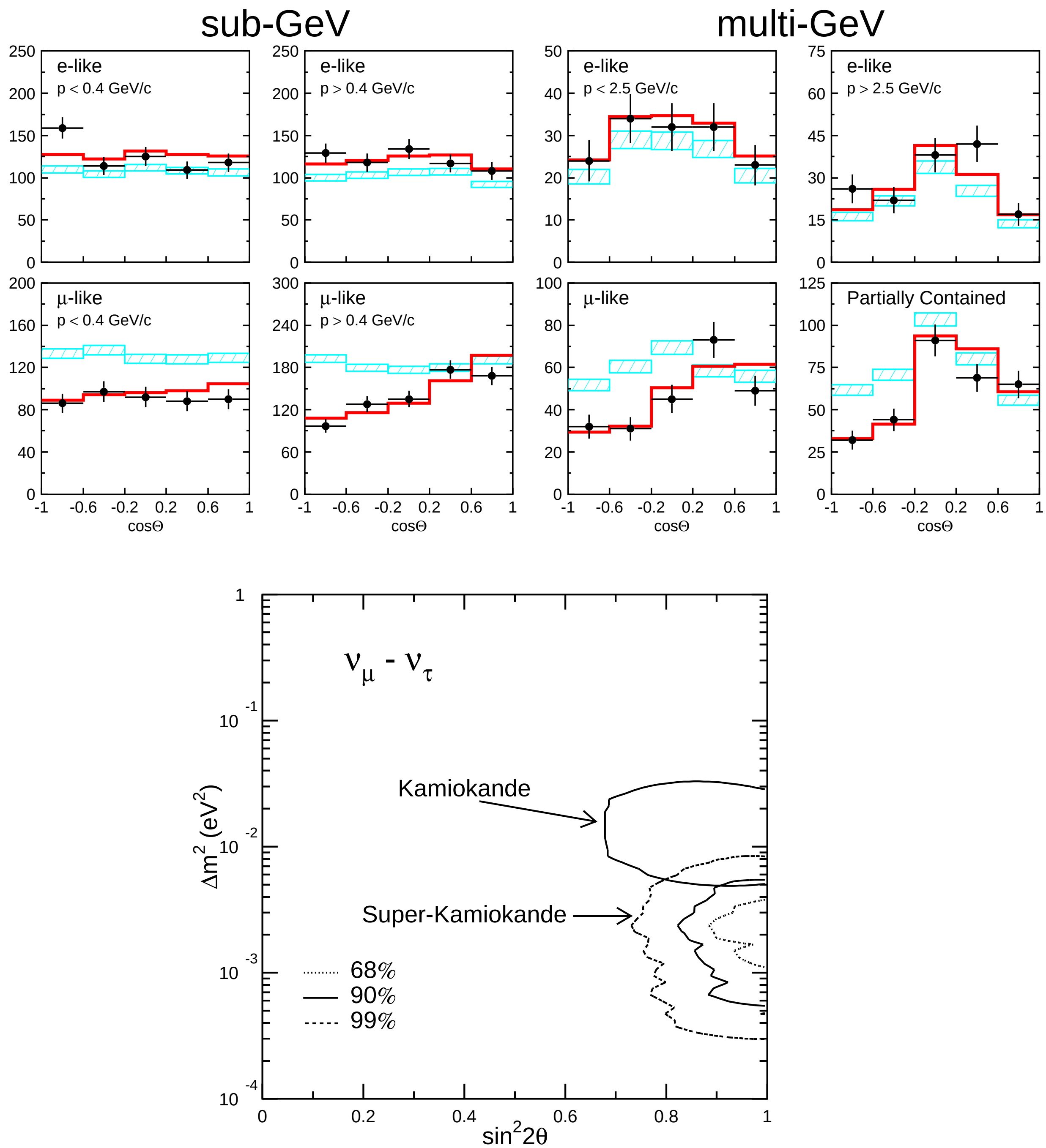

Figure 2.3: Recent results from the Super-Kamiokande experiment and final results from the Kamiokande experiment. Upper (a): Expected and observed zenith angle distributions for Super-Kamiokande sub-GeV and multi-GeV events (shaded bars are the no-oscillation predictions, histograms are the best-fit distributions with oscillations, and points with error bars are the data). Lower (b): Regions of neutrino oscillation parameter space allowed by atmospheric neutrino data from the Super-Kamiokande and Kamiokande experiments.

2.3.3.3 Upward-going muons

In addition to interacting within the fiducial mass of a detector, neutrinos can interact in the rock surrounding the detector. If a rock interaction occurs close enough to the detector, muons resulting from ν_μ charged-current processes can penetrate to the detector and be measured there. Generally, only muons from upward-going neutrinos can be identified as originating from that source, since the flux of muons from downward-going neutrinos is very much smaller than the flux of direct muons (from pion decay in the atmosphere) which penetrates the rock overburden of an underground detector.

The existence of oscillations will affect the results of upward-going muon measurements because oscillations can deplete the ν_μ flux as a function of L/E, and hence of zenith angle. Thus, any discrepancies between predictions and measurements of the rate and zenith angle distribution of upward-going muons can provide information about neutrino oscillation parameters. Detectors to date have not been able to measure the energies of muons unless they stop in the detectors. The ratio of stopping to through-going muons can also be sensitive to the values of oscillation parameters.

The early measurements of upward-going muons performed in the Baksan[40] and IMB[41] detectors showed no evidence of anomalies, but were handicapped by the relatively small sizes of these detectors. More recently, MACRO[42], Kamiokande[43] and Super-Kamiokande[44] have all reported deviations from the predictions, in both the rates and zenith angle distributions. The rate deviations provide less information because the comparison with predictions relies heavily on the knowledge of the absolute normalization of the atmospheric neutrino flux. The zenith angle distributions can provide information which is less sensitive to uncertainties in the theoretical models. The results of all three of these experiments can be explained by neutrino oscillations with the parameters derived from observations of atmospheric neutrino interactions occurring within the detectors themselves, as discussed above.

2.3.3.4 Overview of the ν_{atm} situation

The currently available results on the atmospheric neutrino R-value anomaly are summarized in Figure 2.4. It appears that there is a trend to convergence on a value of R $\sim$ 0.6. The best estimate of the magnitude of the error on this quantity, due mainly to uncertainties in neutrino flux calculations, is about 0.05. Thus the effect appears to be real. In addition, the observed zenith angle dependence of the ν_μ rate favors the oscillation hypothesis as the explanation of this anomaly.

To date, no accelerator experiments have been able to confront these results. On the other hand, two reactor experiments, near Chooz, France, and in Palo Verde, Arizona, have been constructed to test the hypothesis that this anomaly is due to $\nu_\mu \leftrightarrow \nu_e$ oscillations. The CHOOZ experiment has already reported highly significant results[45]: they find no evidence for $\nu_\mu \rightarrow \nu_e$ oscillations with large values of $\sin^2(2\theta)$ and are able to set a 90% CL limit on Δm^2 of 9×10^{-4} eV^2 at $\sin^2(2\theta) = 1$. The result rules out almost all of the Super-Kamiokande suggested region when their data are interpreted under the $\nu_\mu \leftrightarrow \nu_e$ hypothesis.

The situation on the determination of oscillation parameters (assuming the oscillation hypothesis is the correct one) from the zenith angle and L/E distributions, the R-value data and upward going muon results, is still murky. Kamiokande and Super-Kamiokande results

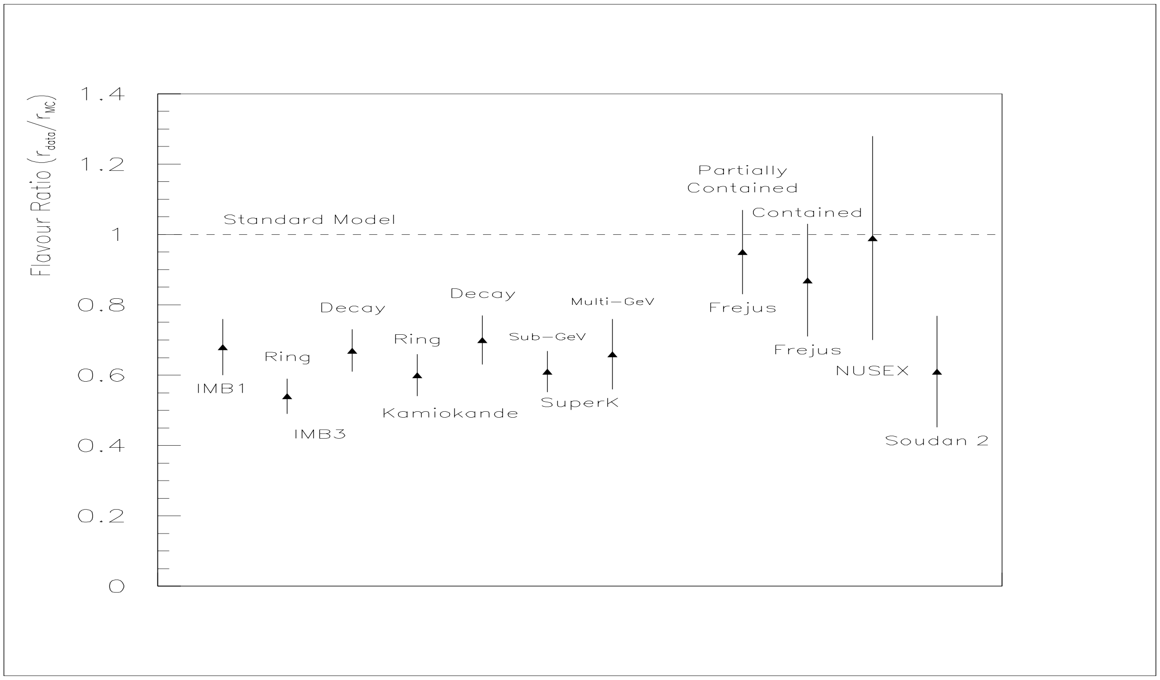

Figure 2.4: Comparison of experimental results for the atmospheric neutrino flavor ratio R. Results from water Cerenkov detectors are on the left and those from solid gas-chamber detectors are on the right.

appear to favor somewhat different values of Δm^2 although they both require values of $\sin^2(2\theta)$ close to unity. The preliminary results reported on this issue by the Soudan 2 Collaboration favor the region of Δm^2 around 10^{-2} eV^2. The recent results from the MACRO, Kamiokande and Super-Kamiokande experiments on upward-going muons are consistent with this full range of Δm^2 values. It is our (conservative) view that a reasonable conclusion from all the data would be

$$\log_{10}(\Delta m^2) = -2.5 \pm 0.5$$

(where Δm^2 is in eV^2) if it is neutrino oscillations which cause the atmospheric neutrino anomaly. Furthermore, the results from CHOOZ and Super-Kamiokande favor $\nu_\mu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_{sterile}$ (or a mixture of the two) as the most likely mode.

2.3.4 The solar neutrino deficit

Because it is unlikely that MINOS can confront directly the question of the solar neutrino deficit, we shall discuss the situation here only very briefly. We know that the nuclear fusion reaction of protons into helium, that is responsible for the production of most of the solar energy, must also generate electron neutrinos, specifically 2 ν's for every helium nucleus made. Thus from the amount of total solar energy generated we can predict the total number of neutrinos created and thus the neutrino flux that should reach the earth.

To obtain the spectrum of these neutrinos, one has to understand the details of the reactions that compose this fusion cycle, as well as the less important CNO cycle. The conventional belief is that the Standard Solar Model[46] is reliable enough that the spectra are understood at the level of a few percent.

The last three and a half decades have seen an extensive experimental effort to detect solar neutrinos and measure their flux. Four major experiments have been mounted and obtained results: Homestake[47], Kamiokande[48], GALLEX[49], and SAGE[50]. They all measure a smaller flux of neutrinos than predicted. Furthermore, because the neutrino energy threshold for detection is quite different in these experiments (except GALLEX and SAGE, which use similar techniques and hence have the same threshold), quantitative analysis of the discrepancies allows one to draw conclusions about any possible energy dependence of the deficit. It appears that the observed depletion does have an energy dependence. Such an effect would be very difficult to generate by a variation of the parameters in the solar model. Hence, one is led to searching for an explanation in the area of particle physics; one of the possibilities would be neutrino oscillations.

There has been an extensive theoretical effort to see how well one can explain the solar neutrino deficiency through the mechanism of neutrino oscillations. A very important contribution in this general area has been by Mikheyev and Smirnov[51] and Wolfenstein[52], who first showed that the different interaction cross sections of the three neutrino species in matter can effectively contribute to neutrino oscillations. These so called "matter oscillations" are able to explain the observed neutrino flux deficit. Two general regions, shown in Figure 2.1 with Δm^2 around 10^{-5} eV^2, appear to fit all of the available data[53]. In addition there is a pure vacuum oscillation solution[54] with a much lower value of Δm^2 (off scale in the Figure) of around 10^{-11} eV^2.

It is unlikely that the solar neutrino anomaly will be illuminated significantly by any terrestrial experiment, except possibly by the proposed KamLAND experiment in Japan, which relies on a very large reactor neutrino detector in the former Kamiokande cavern. The next generation of solar neutrino detectors, e.g., SNO (with its ability to measure the neutral current reaction rate), Super-Kamiokande (with its ability to measure the high energy portion of the flux as well as temporal variations) and Borexino (with its ability to measure the energy spectrum of low energy neutrinos in real time) should, however, be able to shed some new light on this situation.

2.4 New results expected before 2002

We expect that there will be some new experimental information by the year 2002 which will be relevant to the question of possible neutrino oscillations. On the other hand, it is unlikely that definitive answers will exist by that time. We elaborate next on what new information will be available in each of the four areas corresponding to different "hints."

Regarding "dark-matter" neutrinos, it is possible that CHORUS and/or NOMAD will find convincing ν_τ events. On the other hand, their future reach, beyond what is known today, is considerably less than an order of magnitude. Thus it is not clear how convincing the signal would be if they observe the few events that would be allowed by the present limits. In any case, such an observation would not be able to determine the value of Δm^2.

The LSND result should be confirmed or contradicted by 2002. The current LSND run in 1998, if it gives results consistent with the past ones, should determine somewhat better the parameter space of the LSND effect. Equally or even more important is the KARMEN experiment, where an order of magnitude improvement in sensitivity over the past experiment is expected. KARMEN results, which should be available in a year or two, should provide a very strong check on the LSND results.

Several new experimental results will undoubtedly shed some light on the solar neutrino puzzle by 2002. The biggest impact should come from the SNO, Borexino and Super-Kamiokande data. The SNO experiment will be able to measure the neutral current cross section of the solar neutrinos and thus determine the absolute value of the solar neutrino flux. It is a very hard measurement but the initial results should be available by 2002. In addition, they can obtain good total energy measurements for the inverse beta decay events. Super-Kamiokande has the ability to measure the energy spectrum of the neutrinos above 6 MeV and to look for the diurnal and semi-annual variations in the observed interaction rate. Borexino can measure the energy of low energy neutrino interactions as well as the time dependence of the energy spectrum. All of these measurements can provide crucial information that will be able to test the validity of various models attempting to explain the solar neutrino deficit. It is unlikely, in our opinion, that other experiments currently in the planning or construction phase, i.e., ICARUS, the Iodine experiment at the Homestake mine, or the helium detector experiments, will have any significant results by 2002.

As far as the atmospheric neutrino anomaly is concerned, new results are expected in three areas:

a) Both Super-Kamiokande and Soudan 2 will continue their investigations of atmospheric neutrinos. The Super-Kamiokande R value measurements are already beginning to be limited by systematics. Thus additional data will not contribute significantly to a better value of this parameter. The zenith angle distributions, however, will become more informative with a larger data sample.

   The Soudan 2 experiment can make an independent contribution towards understanding the zenith angle distribution and its impact on the estimate of Δm^2. At low energies (below 1 GeV), the major problem in the L/E analysis is not statistical but the smearing of both L and E by the fact that one has to use the outgoing particles to obtain the energy and angle of the interacting neutrino. Both the Fermi motion of the struck nucleon and the presence of unobserved particles smear the calculations of L (from the angle) and of E. Soudan 2 has a potentially significant advantage here over Super-Kamiokande in that recoil protons and low energy charged pions can be observed and measured, thus substantially reducing smearing. The improved L/E resolution of Soudan 2 might well compensate for the poorer statistics. Thus Soudan 2 should be able to test in the future the angular distribution of the interacting neutrinos.

b) The CHOOZ experiment should complete their analysis and the Palo Verde experiment should soon be able to either confirm or cast doubt on the CHOOZ result. Together these experiments should be able to convincingly demonstrate the absence (or otherwise) of $\nu_e \rightarrow \nu_\mu$ oscillations down to values of Δm^2 somewhat below 10^{-3} eV^2 for large mixing.

c) Most important, the K2K experiment[55], which observes neutrinos from KEK with the Super-Kamiokande detector, will be the first accelerator experiment to test the hypothesis that the atmospheric neutrino anomaly is due to $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_{sterile}$. Their calculated sensitivity, expected to be achieved by 2002, for both $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_e$ oscillations, is shown in Figure 2.5.

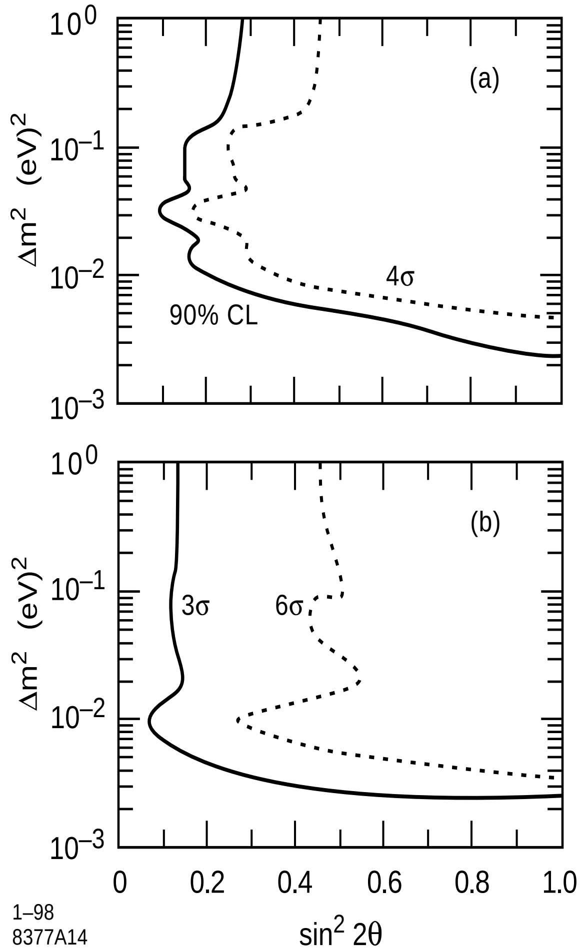

Figure 2.5: Predicted sensitivity of the K2K experiment by the year 2002. Limits on neutrino oscillation parameters are shown for (a) $\nu_\mu \to \nu_x$ (disappearance) and (b) $\nu_\mu \to \nu_e$ (appearance), for the case of no oscillation signal. Note the linear scale in $\sin^2(2\theta)$.

In summary, we list some possible resolutions of the atmospheric neutrino anomaly:

a) The process responsible is $\nu_\mu \to \nu_\tau$ oscillations with Δm^2 in the range 2×10^{-3} to 10^{-2} eV^2. This is the most likely hypothesis if all the relevant existing data are taken at face value. If this is indeed the correct explanation, and Δm^2 is in the upper range, then the results of the K2K experiment should go a long way towards establishing the validity of this hypothesis. In addition, the Soudan 2 L/E distributions and Super-Kamiokande zenith angle measurements would be able to support such a conclusion.

b) The process responsible is $\nu_\mu \to \nu_{sterile}$ with Δm^2 in the range 2×10^{-3} to 10^{-2} eV^2. The existing data do not distinguish between this hypothesis and hypothesis (a) above, but theoretical prejudice has been that (a) is the less radical conjecture. The K2K experiment will provide only very limited information which could discriminate between these two hypotheses.

c) The process responsible is $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_{sterile}$ oscillations with $\Delta m^2 = 2\times10^{-3}$ eV^2 or lower. This possibility appears less likely than the two above when one considers all the available data. If this hypothesis is correct, then we would not expect to see any significant signal from reactor or accelerator experiments until MINOS begins taking data.

Of course, the complete oscillation picture could be more complex. The actual physical situation probably includes contributions from several oscillation modes with different strengths. For completeness, we should also include two other possibilities, even though they do not appear very likely today:

d) The atmospheric neutrino anomaly exists but is unrelated to neutrino oscillations. So far, no satisfactory alternative hypothesis has been put forth to explain the effect, but this does not necessarily exclude this dark-horse possibility.

e) There is no atmospheric neutrino anomaly – the observed effects are either instrumental and/or explainable by modifications of the cosmic ray shower models. Unlikely as this possibility may seem in light of the convergence of most recent results on R = 0.6 and the observation of an up-down asymmetry in atmospheric ν_μ's by Super-Kamiokande, it must be kept in mind if a self-consistent picture for other alternatives cannot be formulated.

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